Adaptive Filter

Primary Input $u(n)$

Reference Input $d(n)$

Adaptive Filter

LMS/RLS

Estimate of Noise $y(n)$

Error Signal $e(n)$

Output $\sum$

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Chapter 0

Course Introduction/Overview

Contents

0.1 Lecture Outline .................................................. 3
0.2 What is this course about? ...................................... 4
0.3 Course Perspective in Comm/DSP Area of ECE .......... 5
0.4 The Role of Computer Analysis/Simulation Tools .... 6
0.5 Instructor Policies ................................................. 7
0.6 Course Syllabus .................................................... 9
0.7 Preview/Overview of Topics ................................... 10
  0.7.1 Linear Filtering ............................................. 10
  0.7.2 Adaptive Filters ........................................... 11
  0.7.3 Filter Structures ........................................... 12
  0.7.4 Approaches to Development ............................ 14
  0.7.5 Additional Comments .................................... 17
  0.7.6 Applications Examples ................................. 17
0.1 Lecture Outline

- What is this course about?
- Course perspective
- The use of computer analysis/simulation tools
- Instructor policies
- Course syllabus
- Background and preview
0.2 What is this course about?

- The focus of this course is estimation theory, in the discrete-time domain
  - Linear optimum filters; e.g., Wiener filters in the discrete-time domain
  - Linear prediction
- Linear adaptive filters
  - Least-mean-square (LMS) algorithm
  - Recursive least-squares (RLS) based algorithms
  - Others
- Applications
  - Adaptive equalization
  - Speech coding
  - Spectrum analysis
  - Adaptive noise/interference cancellation
  - Adaptive beamforming
  - Adaptive control
0.3 Course Perspective in Comm/DSP Area of ECE

Communications/DSP Course Offerings

- Intro to DSP
- Comm Sys I
- Random Signals
- Modern DSP
- Comm Sys II
- Wireless/ Mobile Comm
- Detect/ Estim.
- Real Time DSP
- Spread Spectrum
- Optical Comm
- Radar Systems
- Image Proc
- Comm Topics
- Spectral Estim.
- Opt Comm
- Comm Networks
- Satellite Comm
- Defined MSEE Courses
- UCCS Senior/ 1st Tier Grad. Signals & Systems Courses
- UCCS 2nd Tier Grad. Signals & Systems Courses
- UCCS Other Grad. Signals & Systems Courses – On Demand/Ind. Study
- Linear Systems
- Prob. & Rand. Var
- Undergraduate Electrical Engineering Curriculum

ECE 6650 Estimation Theory and Adaptive Filtering
0.4 The Role of Computer Analysis/Simulation Tools

- In working homework problems pencil and paper type solutions are mostly all that is needed
  - It may be that problems will be worked at the board by students
  - In any case pencil and paper solutions are still required to be turned in later
- Occasionally an analytical expression may need to be plotted, here a visualization too such as MATLAB or Mathematica will be very helpful
- Simple simulations can be useful in enhancing your understanding of mathematical concepts
- The use of MATLAB for computer work is encouraged since it is fast and efficient at evaluating mathematical models and running Monte-Carlo system simulations
- There will be one or more MATLAB based simulation projects, and the Haykin text supports this with MATLAB based exercises
0.5 Instructor Policies

- Working homework problems will be a very important aspect of this course
- It may be that problems (some) will be worked in class by students
  - If problems are assigned to be worked on the board, please come to class prepared
  - If you get stuck on some aspect of a board problem still plan to present the problem in part at the intended class meeting
  - We (the whole class) will work together at getting the trouble areas straight so that the problem can be completed after class
  - All problems are finally to be worked in paper form by all students, and will be due at some time following the in-class working of the problems
- If work travel keeps you from attending class on some evening, please inform me ahead of time so I can plan accordingly, and you can make arrangements for turning in papers
- The course web site
  http://eceweb.uccs.edu/wickert/ece6650/
  will serve as an information source between weekly class meetings
- Please check the web site updated course notes, assignments, hints pages, and other important course news; particularly on days when weather may result in a late afternoon closing of the campus
0.6 Course Syllabus

ECE 6650
Estimation Theory and Adaptive Filtering
Spring Semester 2005

Instructor: Dr. Mark Wickert
Office: EB-226
Phone: 262-3500
Fax: 262-3589
wickert@eas.uccs.edu
http://eceweb.uccs.edu/wickert/ece6650/

Office Hrs: Thurs. 2:15–3:00 pm, others by appointment. Note: These hours may be adjusted if needed.


Optional Software: MATLAB Student Version 7.x, release 14. An interactive numerical analysis, data analysis, and graphics package for Windows/Mac/Linux $99.95$. The Signal Processing toolbox will also be needed $29.95$. Order both from [www.mathworks.com/student](http://www.mathworks.com/student). Note: The ECE PC Lab has the full version of MATLAB and Simulink for windows (ver. 7.x) with many toolboxes.

Grading:
1.) Graded homework assignments, both Problems and Computer-oriented Problems worth 60%.
2.) Midterm exam, most likely a take-home computer simulation problem, worth 20%.
3.) Final exam or project worth 20%.

<table>
<thead>
<tr>
<th>Topics</th>
<th>Text Sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>0. Background and Preview</td>
<td>0.1–0.8</td>
</tr>
<tr>
<td>1. Stochastic Processes and Models</td>
<td>1.1–1.17</td>
</tr>
<tr>
<td>2. Introduction to Estimation Theory</td>
<td>App. D &amp; notes</td>
</tr>
<tr>
<td>3. Wiener Filters</td>
<td>2.1–2.9</td>
</tr>
<tr>
<td>4. Linear Prediction</td>
<td>3.1–3.11</td>
</tr>
<tr>
<td>5. Method of Steepest Descent</td>
<td>4.1–4.6</td>
</tr>
<tr>
<td>7. Normalized Least-Mean-Square Adaptive Filters</td>
<td>6.1–6.5</td>
</tr>
<tr>
<td>8. Frequency-Domain and Subband Adaptive Filters</td>
<td>7.1–7.7</td>
</tr>
<tr>
<td>9. Method of Least Squares</td>
<td>8.1–8.15</td>
</tr>
<tr>
<td>10. Recursive Least-Squares Adaptive Filters</td>
<td>9.1–9.9</td>
</tr>
<tr>
<td>11. Kalman Filters</td>
<td>10.2–10.10</td>
</tr>
<tr>
<td>12. Square-Root Adaptive Filters</td>
<td>11.1–11.5</td>
</tr>
<tr>
<td>13. Order-Recursive Adaptive Filters</td>
<td>12.1–12.14</td>
</tr>
<tr>
<td>15. Tracking of Time-Varying Systems</td>
<td>14.1–14.9</td>
</tr>
</tbody>
</table>
0.7 Preview/Overview of Topics

0.7.1 Linear Filtering

- When a signal is filtered we typically view the filters operation as that of extracting the information of interest and leaving behind a noise or interference signal.

- A filter can be used to perform:
  1. Filtering, the desired output is obtained using the present and past values of the input.
  2. Smoothing, the desired output is obtained using past, present, and future input values; clearly the use of future input values introduces a delay in producing the desired output values.
  3. Prediction, the quantity of interest is a forecast into the future using present and past input data.

- Since the filter is assumed linear the filter output must be a linear combination of the applied inputs.

- In statistical filter design we may for example assume that we have knowledge of the mean and autocorrelation function of the relevant signals (e.g. desired and noise); the filter is then designed to minimize some statistical criterion.

- The Wiener filter assumes stationary statistics and minimizes the mean-square error between the desired response and the actual filter output.

- For nonstationary signals, then a time-varying filter is needed; the Kalman filter is also possible solution.
0.7.2 Adaptive Filters

- The Wiener filter requires a priori knowledge of the data statistics.
- If the statistical information is incomplete the filter may first need to estimate the statistical parameters of interest, and then “plug” them into a formula that computes the desired filter parameters.
- The above procedure does not work well for real-time operations.
- An adaptive filter is self-designing in the sense that it uses a recursive algorithm to continuously adjust the filter parameters with only limited knowledge of the signal characteristics.
- In a stationary environment the adaptive filter will converge to the optimum Wiener filter.
- In a nonstationary environment the adaptive filter can track variations in the signal statistics provided they are not changing too rapidly.
- Many recursive algorithms for adaptive filters have been developed, each has advantages and disadvantages.
- Algorithm performance factors include:
  - Rate of convergence
  - Misadjustment
  - Tracking
  - Robustness
  - Computational requirements
  - Structure
0.7. PREVIEW/OVERVIEW OF TOPICS

– Numerical properties

0.7.3 Filter Structures

Transversal or FIR

\[
y(n) = \sum_{k=0}^{M} w_k^* u(n - k)
\]

Lattice Predictor

- The signals \( f_m(n) \) and \( b_m(n) \) are the forward and backward prediction errors respectively
Recursive or IIR

- With feedback we now have the potential for instability
- FIR adaptive filters are far more popular for this reason

0.7.4 Approaches to Development

There are three distinct approaches that will be considered:

1. Stochastic gradient
2. Least-squares
Stochastic Gradient Approach

- The filter structure is a tapped delay line
- The mean-square error is minimized with the resulting error surface being a multidimensional paraboloid with a unique minimum point
- The algorithm, which uses the method of steepest descent, is known as the least-mean square (LMS) algorithm
- The tap weight vector is adjusted as follows:

\[
\begin{pmatrix}
\text{updated value} \\
\text{of tap-weight vector}
\end{pmatrix} = \begin{pmatrix}
\text{old value} \\
\text{of tap-weight vector}
\end{pmatrix} + \begin{pmatrix}
\text{learning-rate parameter} \\
\text{tap-input vector}
\end{pmatrix} \begin{pmatrix}
\text{error signal}
\end{pmatrix}
\]

- In a nonstationary environment the error-performance surface changes continuously, so the LMS must continually track the bottom of this surface
- Changes in the input statistics must be slow compared with the LMS learning rate for tracking to occur

Least-Squares Estimation

- Here the adaptive algorithm is based on the method of least squares; a performance index consisting of the weighted sum of the squared error is minimized
- The formulation may be either block estimation or recursive estimation
A special form of Kalman filtering where a state is updated

\[
\begin{pmatrix}
\text{updated value of the state} \\
\end{pmatrix}
= \begin{pmatrix}
\text{old value of the state} \\
\end{pmatrix}
+ \begin{pmatrix}
\text{Kalman gain} \\
\text{innovation vector} \\
\text{error signal}
\end{pmatrix}
\]

where the innovation vector represents new information into the filter process.

Three basic classes of recursive least squares algorithms exist:

- **Standard Recursive least-squares (RLS)**: The algorithm relies on the matrix-inversion lemma; the algorithm converges rapidly and has rather high computational complexity; lack of numerical robustness

- **Square-root RLS**: Two processing stages required (1) orthogonal triangularization of the input data matrix; (2) computation of the least squares weight vector (\(QR\)-decomposition); numerically robust; in a matrix sense a square-root form of the standard RLS

- **Fast RLS**: The complexity of the first two RLS forms is \(O(M^2)\), while for the fast RLS just \(O(M)\), where \(M\) is the filter order; not always numerically stable

The RLS class is noted for rapid convergence.
0.7.5 Additional Comments

- The LMS class of algorithms is said to be *model independent*, so they exhibit good tracking performance.
- The RLS class is *model dependent*, and hence tracking is more sensitive to mismatch between the mathematical model and the true physical process producing the input.
- When choosing an adaptive filter practical issues that are important are:
  - computational cost
  - performance
  - robustness
- Adaptive filter algorithms generally assume the input data is in baseband form, for bandpass signals this means complex baseband following frequency translation

\[ u(n) = u_I(n) + j u_Q(n) \]

Algorithms are thus typically developed in *complex form*, with the *real form* being a special case.

- The use of computer simulation is very useful as a first step in the evaluation process.

0.7.6 Applications Examples

Adaptive filters find application in communications, control, radar, sonar, seismology, image processing, and pattern recognition. The following examples assume the data is in baseband form, thus the data may be real valued or complex. We begin with four basic classes.
CHAPTER 0. COURSE INTRODUCTION/OVERVIEW

Identification

- Provides a best fit linear model to the unknown plant

Inverse Modeling

- Here the adaptive filter ideally corresponds to the transfer function inverse of the unknown plant

Prediction

- The adaptive filter uses a linear combination of past inputs to form an estimate of the present value of the random signal input
Interference Cancelling

- The adaptive filter is used to cancel unknown interference from a desired information carrying signal.
- Here a reference signal is employed which typically is chosen to have little or no information signal component.

Example 0.1: **System Identification**
Example 0.2: **Adaptive Equalization**

Data transmission with no equalization

Example 0.3: **Digital Representation of Speech**

Model of Speech Production Process
0.7. PREVIEW/OVERVIEW OF TOPICS

Linear Predictive Coding (LPC) (a) transmitter (b) receiver

Waveform Coding (a) PCM, (b) DPCM, (c) ADPCM
Example 0.4: **Autoregressive Spectrum Analysis**

The power spectrum of an AR process is given by

\[
S_{ar}(\omega, n) = \frac{\sigma_v^2}{|1 + \sum_{k=1}^{M} a_k^*(n) e^{j k \omega}|^2}
\]

---

Example 0.5: **Adaptive Detection**

- \(H_1\): Colored Noise + White Noise
- \(H_2\): Signal + Colored Noise + White Noise
• Preprocess using an *adaptive line enhancer* (ALE)

**Example 0.6: Adaptive Noise Cancellation**
Example 0.7: Adaptive Beamforming

With adaptive beamforming the filtering switches from the time domain (temporal) to the spatial domain. Sensors must be spaced appropriately to avoid grating lobes. A linear array of sensors is analogous to a transversal or FIR filter. Applications include:

- Communications (smart antennas for space division multiple access and interference rejection)
- Radar (source angle of arrival)
- Sonar (hydrophone arrays)
- Speech enhancement
0.7. PREVIEW/OVERVIEW OF TOPICS

Five sensor adaptive beamformer
Linear array geometry for an incident plane wave
• Wireless LAN (WLAN) study at UCCS

Multiple beam processors at a WLAN access point

The generalized sidelobe canceller with arrival angle estimation
CHAPTER 0. COURSE INTRODUCTION/OVERVIEW

Overlay of Antenna Patterns

Sample pattern in engineering building

20 ft

Trajectory of Moving User (Path 2)

Access Node with 5 Element Array

Fixed User (User 2)

Beam Pattern Adapted to Fixed User

Beam Pattern Adapted to Moving User

Moving User

Southwest Wing of Engineering

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