Due Friday December 8, 2017

Problems:

1. A PLL with loop filter is designed to track a carrier in additive noise, in a coherent communications system. The operating parameters are chosen such that $\zeta = 1/\sqrt{2}$, $r_2/r_1 = 0.002$, and $KA = KPKv = 6000$ (note this is a passive loop filter with $K_{LF} = 1$, see notes 4–40). Assume that the loop phase error $\phi(t)$ is small so the linear noise model is valid.

   (a) Find the time constants $r_1$ and $r_2$.
   (b) Find the single-sided loop noise equivalent bandwidth $B_n = B_L$.
   (c) If $N_0 = -161$ dBm/Hz, find the received carrier power $P_c$, which yields $\sigma_\phi^2 = 0.1$ rad$^2$ using the linear noise theory.

2. (Similar to Egan 12.1 & 12.2) Shown in the figure below is the one-sided phase noise power spectral density at the input to the PLL also shown below.

\[ S_{\phi}(f) = S_{\phi}(f) \]

\[ K_P = 1V/\text{rad} \]

\[ K_v = 625\text{Hz/v} \]

\[ S_{\phi,\text{in}}(f) = S_{\phi}(f) \]

\[ S_{\phi,\text{out}}(f) = S_{\phi}(f) \]
2. (cont.)

(a) Develop a phase noise model which corresponds to the phase noise plot shown above, of the form

\[ S_{\phi, r}(f) = S_{\phi, f}(f) = k_{-3} \frac{1}{f^3} + k_{-2} \frac{1}{f^2} + k_0 \]

Note that the units in the figure are dBr/Hz, where dBr stands for dB-rad^2, which makes it the same quantity as \( S_{\phi, f}(f) \) in the notes.

(b) Find the one-sided phase noise spectral density at test point A and plot it in dB versus log frequency.

(c) Find the one-sided phase noise spectral density at test point B and plot it in dB versus log frequency.

(d) What is the variance of the closed-loop phase error, \( \sigma_{\phi}^2 \), due to the input phase noise, over just the band of frequencies 100 Hz to 200 kHz. A numerical solution to the integral is sufficient.

3. (Egan 8.1 modified) A loop has a double-balanced mixer phase detector with a (maximum) gain of \( K_p = 0.1 \) V/cycle. The tuning characteristic of the oscillator has a 40-MHz/V slope. The loop filter is shown below. The VCO is 90° out of phase with an input signal to which it is locked when the frequency is 30 MHz (note this is a locked 0° phase error condition).

(a) If the input frequency drifts, how high can it go before the loop will lose lock? Give frequencies in Hz.

(b) After lock is broken, the input frequency is lowered again. At what frequency can lock be reacquired? What are the restrictions on the formula that you used and how well are they met (give numerical values)?

(c) At what difference between input frequency and VCO center frequency \( f_c \) will cycle skipping stop? What are the restrictions on the formula that you used and how well are they met (give numerical values)?

(d) How long will it take to stop cycle skipping if the input signal is 14 kHz above \( f_c \)? What are the restrictions on the formula that you used and how well are they met (give numerical values)?
4. Consider the notes Chapter 4 section on optimum loop transfer functions. Let $\theta(t) = [\theta_0 + \Omega_0 t]u(t)$ be a composition of phase step and phase ramp modulating the input carrier along with additive noise $n(t)$. It can be shown that the optimum loop transfer function is (I have made this a bonus problem in the past, no worries about proving here):

$$H_{opt}(s) = \frac{2\zeta \omega_n s + \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}$$

where

$$\omega_n = \sqrt{\lambda \Omega_0} \left( \frac{2A^2}{N_0} \right)^{1/4} \quad \text{and} \quad \zeta = \frac{1}{2} \sqrt{2 + \frac{\theta_0^2}{\Omega_0} \sqrt{\frac{2A^2}{N_0}}}$$

(a) If $P_c/N_0 = 17$ dB-Hz, $\theta_0 = \pi/9$ rad, and $\Omega_0 = 2\pi \times 10$ rad/s, what are $B_n = B_L$, $\zeta$, and $\sigma_\phi^2$?

(b) Repeat part (a) with $P_c/N_0 = 30$ dB-Hz, $\theta_0 = \pi/4$ rad, and $\Omega_0 = 2\pi \times 100$ rad/s.

(c) In part (b) if $\theta_0$ is taken to be zero, what is the error in $B_n = B_L$ and $\zeta$ in percent over the actual optimum?

5. One more?