Problems:

1. A TBD problem on sinusoidal phase/frequency modulation.

2. In this problem you will compare the transient response of the Type II APLL with the corresponding Type II DPLL. From notes Chapter 3, the closed-loop system functions are given by

\[
H(s) = \frac{K (\tau_2 s + 1)}{s^2 + K \frac{\tau_2}{\tau_1} s + \frac{K}{\tau_1}} = \frac{2\zeta \omega_n s + \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}
\]

\[
H(z) = \frac{k_1 z^{-1} + (k_2 - k_1) z^{-2}}{1 - (2 - k_1) z^{-1} + (1 - k_1 + k_2) z^{-2}}
\]

For the continuous-time system

\[
\omega_n = \sqrt{\frac{K}{\tau_1}}, \quad \zeta = \frac{1}{2\omega_n} K \frac{\tau_2}{\tau_1} = \frac{\tau_2}{2} \sqrt{\frac{K}{\tau_1}}, \quad B_n = \frac{\omega_n}{2} \left( \zeta + \frac{1}{4\zeta} \right).
\]

For the discrete-time system, with noise bandwidth \(B_n\) in Hz much less than the sampling rate \(f_s\), \(\omega_n\) and \(\zeta\) for the continuous and discrete-time systems are related by

\[
k_1 = k_c k_d \alpha_1 = \frac{2\zeta \omega_n}{f_s} + \frac{1}{2} \left( \frac{\omega_n}{f_s} \right)^2
\]

\[
k_2 = k_c k_d \alpha_2 = \left( \frac{\omega_n}{f_s} \right)^2.
\]

(a) Choose \(B_n = 10\) Hz and \(\zeta = 1/\sqrt{2}\) as the point of comparison with \(f_s = 1000\) Hz. Plot comparison curves for both the phase step response and the frequency step response. Assume the linear model is valid.

(b) Repeat part (a) with \(f_s = 100\) Hz.

(c) Repeat part (a) with \(f_s = 10000\) Hz.
3. Construct in Python a baseband DPLL simulation framework making use of the classes DDS and accum described below for both the phase controller and the loop filter, and that uses a selectable linear or sin() phase detector, i.e.,

\[ \phi[n] = \sin (\theta[n] - \theta[n]) \]
or
\[ \simeq \theta[n] - \theta[n] \]

(a) Test the simulation in the linear and nonlinear modes verifying that for a small frequency step the response matches theory as developed in the notes/book/and problem (1) above. Choose \( B_n = 10 \text{ Hz} \) and \( \zeta = 1/\sqrt{2} \).

(b) Repeat the test above, except now use a frequency the step size that causes one or more cycle slips before the loop pulls back in. Verify also that the cycle slipping only occurs in the nonlinear mode.

(c) Extend the baseband model of (a) to a complex baseband simulation model that takes input \( e^{j\phi[n]} \) uses the DDS class as the phase controller since it outputs \( e^{-j\hat{\theta}[n]} \). The phase detector will as in the DPLL lab output \( \phi[n] = \text{Im}\{e^{j\theta[n]} \cdot e^{-j\hat{\theta}[n]}\} \). Verify that the complex baseband model performs similar to the baseband model, when in the nonlinear mode.

Classes:

class DDS(object):
    ""
    Implementation of a DDS that outputs \( \exp(-1j*\theta\_hat) \)
    
    Mark Wickert October 2017
    ""

def __init__ (self,fcenter,fs,kc = 1,state_init = 0):
    ""
    Initialize the DDS with a center frequency in Hz, the sampling rate, the gain kc, and initial theta\_hat state in radians.
    ""
    self.fcenter = fcenter
    self.fs = fs
    self.delta_w = 2*pi*self.fcenter/self.fs
    self.kc = kc
    self.theta\_hat = state_init

def update(self,e_in):
    ""
    Update the DDS phase accumulator
    ""
self.theta_hat += self.delta_w + self.kc*e_in
if self.theta_hat >= 2*pi:
    self.theta_hat -= 2*pi
elif self.theta_hat < 0:
    self.theta_hat += 2*pi

def output_exp(self):
    """
    Output exp(-j*theta_hat)
    """
    return exp(-1j*self.theta_hat)

def set_fcenter(self,fcenter_new):
    """
    Set a new center frequency in Hz
    """
    self.fcenter = fcenter_new
    self.delta_w = 2*pi*self.fcenter/self.fs

class accum(object):
    """
    An accumulator class for use in DPLL loop filters
    """
    Mark Wickert October 2017
    """
    def __init__(self,state=0):
        """
        Initialize the accumulator object by setting the state
        """
        self.state = state

    def update(self,x_in):
        """
        Update the accumulator
        """
        self.state += x_in

Note: In modeling a complex baseband PLL, the NCO quiescent frequency, f_center should be set to zero. Non-zero values are used the DPLL lab.
4. A PLL with loop filter is designed to track a carrier in additive noise, in a coherent communications system. The operating parameters are chosen such that $\zeta = 1/\sqrt{2}$, $\tau_2/\tau_1 = 0.002$, and $KA = K_p K_v = 6000$ (note this is a passive loop filter with $K_{LF} = 1$, see notes 4–40). Assume that the loop phase error $\phi(t)$ is small so the linear noise model is valid.

(a) Find the time constants $\tau_1$ and $\tau_2$.
(b) Find the single-sided loop noise equivalent bandwidth $B_L$.
(c) If $N_0 = -161$ dBm/Hz, find the received carrier power $P_c$, which yields $\sigma_\phi^2 = 0.1 \, \text{rad}^2$ using the linear noise theory.