Set #1

Due Wednesday February 5, 2020

Make note of the following:
- We will begin discussion of these problems the first class meeting
- Write only on one side of the paper
- Please try if possible to start each new problem on a clean sheet of paper
- Use engineering paper if you like

Problems:

1. (Egan 2.2) In the linear loop model shown below,
\[ \theta(t) = \varphi_{in}(t) \]
\[ K_p \quad K_{LF} \quad K_v \quad \frac{1}{s} \]
\[ \theta(t) = \varphi_{out}(t) \]

we have \( K_p = 0.1 \text{ V/c}, \ K_{LF} = 3, \text{ and } K_v = 1 \text{ MHz/V} \) (note c denotes cycles and there are \( 2\pi \) radians per cycle).

(a) Show that the unity open-loop gain occurs at 47.75 kHz.

(b) If \( K_p \) is changed to 1 V/rad, what is the unity-gain frequency (don’t forget to include units)?
2. (Egan 3.3) In the passive loop filter model shown above let $R_1 = 1000 \ \Omega$, $R_2 = 100 \ \Omega$, and $C = 1 \ \mu F$.

   (a) Write $F(s)$ for this filter.
   (b) What are the gain and phase shift at 500 Hz?

3. (Egan 3.4) For the following loop filter circuits let $R_1 = 1 \ \text{k}\Omega$ and $G_a$ be very large.

   (a) In the loop filter shown below what are the values of $R_2$ and $C$ to give a high frequency gain of 5 and a 1 Hz gain of 100?

   ![Loop Filter Diagram]

   (b) What are the values of the components of the loop filter shown below to give a pole at 20 Hz, a zero at 100 Hz, and a DC gain of 100?

4. Derive an exact relationship between the phase margin, $\phi_m$, and the damping factor, $\zeta$, for the following second-order PLLs. Note that exact means that you must use the exact open-loop gain frequency, $\omega_c$, which gives $|G_0(j\omega)| = 1$ to find $\phi_m$.

   (a) $F(s) = \frac{1+sz}{s^2}$. Hint: In the 4th-order polynomial involving $\omega_c$ let $\gamma = \omega_c^2$ and find the real and positive solution. You now have an expression for $\omega_c$ to put into an expression for $\phi_m$ as a function of $\phi_m(\omega_c)$, i.e. $\phi_m(\omega_c)$ but $\omega_c$ is a function of $\zeta$.

   (b) $F(s) = \frac{1}{1+sz}$ (optional for 10 bonus points).

   (c) $F(s) = \frac{1+sz}{1+sz}$ (optional for 10 bonus points).