**Example:** Consider two finite length sequences

\[ y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n - k] \]

- The sum must be evaluated for each value of \( n \)

- The first nonzero value occurs when \( n - n_3 = n_1 \) or \( n = n_1 + n_3 \)
- The last nonzero value occurs when \( n - n_4 = n_2 \) or \( n = n_2 + n_4 \)
Summary: $y(n)$ is nonzero for at most

$$n_1 + n_3 \leq n \leq n_2 + n_4 \Rightarrow N_y = N_x + N_h - 1$$

Example: Two finite duration sequences given by

$$x[n] = h[n] = u[n] - u[n - N], \quad N > 0$$

- From the previous example we know that $y[n]$ will be at most nonzero on the interval $[0, 2(N - 1)]$
- There are 4 cases to consider:

Case 1: $n < 0$ which implies no overlap, so $y[n] = 0$

Case 2: $0 \leq n \leq N - 1$

$$y[n] = \sum_{k=0}^{n} (1)(1) = n + 1$$
– Case 3: $N - 1 \leq n \leq 2N - 2$

$$y[n] = \sum_{k=n-(N-1)}^{N-1} (1)(1) = (N - 1) - (n(N - 1)) + 1$$

– Case 4: $n > 2N - 2$ which implies no overlap, so $y[n] = 0$

– In summary:

$$y[n] = \begin{cases} 
    n + 1, & 0 \leq n \leq N - 1 \\
    2N - 1 - n, & N - 1 \leq n \leq 2N - 2 \\
    0, & \text{otherwise}
\end{cases}$$
**Example:** Two finite duration sequences in sequence explicit representation:

\[ h[n] = \{1, 2, 1, -1\}, \quad x[n] = \{1, 2, 3, 1\} \]

- In the above notation the arrows indicate where \( n = 0 \)
- We need to evaluate the convolution sum for \(-1 \leq n \leq 5\)
- To evaluate construct the following table:

<table>
<thead>
<tr>
<th>( h[n - k] )</th>
<th>0</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>1</th>
<th>0</th>
<th>0</th>
<th>y[n]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n = -1 )</td>
<td>-1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( n = 0 )</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>( n = 1 )</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>( n = 2 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>( n = 3 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>( n = 4 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>2</td>
<td>-2</td>
</tr>
<tr>
<td>( n = 5 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
</tr>
</tbody>
</table>

- The final output is thus

\[ y[n] = \{...0, 1, 4, 8, 8, 3, -2, -1, 0, 0,...\} \]

- Is this reasonable? The output should start at \((-1 + 0) = -1\) and should stop at \((3 + 2) = 5\), which is indeed the case
**Example:** Two infinite duration sequences,

\[ x[n] = u[n], \quad h[n] = a^n u[n] \]

- By direct substitution into the convolution sum formula we have

\[ y[n] = \sum_{k=-\infty}^{\infty} a^k u[k] u[n-k] \]

- The term \( u(k) \) sets the lower sum limit to zero while the term \( u(n-k) \) sets the upper sum limit to \( n \), hence

\[ y[n] = \begin{cases} 
0, & n < 0 \\
\sum_{k=0}^{n} a^k, & n \geq 0 
\end{cases} = \frac{1 - a^{n+1}}{1 - a} u[n] \]