Final Exam

Due Thursday May 16, 2019 at 12 pm

The final exam is in the form of a take-home exam, meaning each student is to due his/her own work without consulting others. The problems are taken from the Hayes text and elsewhere. The due date 12:00 PM May 16, 2019.

Problems:


2. **Spectral Estimation**: Hayes Text computer exercise C8.12 a–c. In part (b) do not repeat the estimation of the signal powers using the correct frequency. In part (c) only consider the MUSIC algorithm. See the Jupyter notebook Music_helper.ipynb for support code. In part (a) it is OK to let the number of samples $N$ used to estimate $r_x(k)$ be 10,000, to get good estimates. In part (c) you should get very good results with just $N = 1000$ or less. You are free to take $N = 10,000$ if you wish.

3. **Kalman Filtering**: Extend notes Example 8.8. We now assume that the acceleration $x_a(n)$ is an AR(1) process rather than a white noise process. Let $x_a(n)$ be given by

$$x_a(n) = ax_a(n-1) + v(n), \quad v(n) \sim N(0, \sigma_v^2), \quad x_a(0) = 0$$

a.) Augment the state vector $x(n)$ in notes Example 8.8 (page 8-58), with variable $x_a(n)$, and develop the process model as well as the observation model, assuming that only the position is measured.

b.) Using the above model and the parameter values

$$T = 0.1, \quad \alpha = 0.9, \quad \sigma_v^2 = \sigma_w^2 = 0.25,$$ $$x_p(0) = 0, \quad x_v(0) = 1$$

simulate the linear motion of the object. Using the Kalman filtering equations, estimate the position, velocity, and acceleration values of the object at each $n \geq 1$. Generate performance plots similar to the ones given in Notes Example 8.8. A Jupyter notebook zip Kalman.zip is available on the Web site.

c.) Now assume that noisy measurements of $x_v(n)$ and $x_a(n)$ are also available, that is, the observation model is

$$y(n) = \begin{bmatrix} y_p(n) \\ y_v(n) \\ y_a(n) \end{bmatrix} = \begin{bmatrix} x_p(n) \\ x_v(n) \\ x_a(n) \end{bmatrix} + w_1(n)$$

where $w_1(n)$, $w_2(n)$, and $w_3(n)$ are independent zero-mean white Gaussian noise sources, each with variance $\sigma_w^2$. Repeat parts (a) and (b) above.