Phasor Addition Example #1

• Consider the signal
  
  \[ x(t) = 87 \cos(2\pi \cdot 880 \cdot t - \pi/4) \]
  
  \[ + 12 \text{Re}\{(-3 - j4)e^{j(2\pi \cdot 880 \cdot t + \pi/6)}\} \]
  
  \[ + 28 \sin(2\pi \cdot 880 \cdot t - \pi/8) \]

• Find \( X = A e^{j\theta} \) such that \( x(t) = A \cos(2\pi \cdot 880 \cdot t + \theta) \)

By inspection

\[ X = 87e^{-j\pi/4} + 12(-3 - j4)e^{j(\pi/6)} + 28e^{-j(\pi/8 + \pi/2)} \]

• Note \( \sin(\theta) = \cos(\theta - \pi/2) \)

• To obtain a numerical solution for \( X \) we may simply enter values into a calculator

\[
\begin{align*}
87 \cdot e^{-\frac{j\pi}{4}} &+ 12(-3 - j4) \cdot e^{\frac{j\pi}{6}} \\
43.6262 - 146.956i \\
\end{align*}
\]

• Working out more of the lower level steps, we can start by writing

\[ X = 87\{\cos(-\pi/4) + j\sin(-\pi/4)\} \]

\[ + 12\{-3 - j4\}\{\cos(\pi/6) + j\sin(\pi/6)\} \]

\[ + 28\{\cos(-5\pi/8) + j\sin(-5\pi/8)\} \]

• Evaluating the cos and sin terms we have
\[ X = \{61.518 - j61.518\} + \{-7.177 - j59.569\} + \{-10.715 - j25.869\} = \{-43.626 - j146.956\} = 153.295e^{-j1.282} \]

- The direct calculation and the indirect calculation are in agreement
- Here a Mathematica plot (could also have been MATLAB) of the actual time domain waveform is used to experimentally determine \( A \) and \( \phi \)

\[
\text{Plot}\left[87 \cos(2\pi 880 t - \pi / 4) + 12 \text{Re}\left((-3 - j4) e^{j(2\pi 880 t + \pi/6)}\right) + 28 \sin(2\pi 880 t - \pi / 8)\right], \{t, 0, 2 / 880\}, \text{PlotStyle} \to \text{Thick}, \text{AxesLabel} \to \{"t", "x(t)"}\]

The composite waveform

\[
\text{FindMaximum}\left[87 \cos(2\pi 880 t - \pi / 4) + 12 \text{Re}\left((-3 - j4) e^{j(2\pi 880 t + \pi/6)}\right) + 28 \sin(2\pi 880 t - \pi / 8)\right], \{t, 0.0002\}\]

\( \{153.295, \{t \to 0.000231892\}\} \)

\( \phi = -2\pi \left(\frac{0.00023189169947837138}{1 / 880}\right) \)

\( -1.28218 \)
Phasor Addition Example #2

- Consider the signal
  \[ x(t) = 30 \cos(2\pi f_0 t - \pi/2) + B \cos(2\pi f_0 t + \phi) + 60 \sin(2\pi f_0 t + \pi/4) \]
- Find \( X_B = Be^{j\phi} \) such that \( x(t) = 50 \cos(2\pi f_0 t + \pi/4) \)

- We observe that this is just a variation on the previous problem
- We start by writing
  \[
  X = 50e^{j\pi/4}
  = 30e^{-j\pi/2} + Be^{j\phi} + 60e^{j(\pi/4 - \pi/2)}
  \]
  so
  \[
  X_B = Be^{j\phi} = 50e^{j\pi/4} - [30e^{-j\pi/2} + 60e^{-j\pi/4}]
  \]
- Via a TI-89

\[ B = 108.013 \quad \phi = 1.636 \text{ rad} \]
• A lower level analysis can also be performed, as with #1

\[
X_B = 50 \left[ \cos \left( \frac{\pi}{4} \right) + j \sin \left( \frac{\pi}{4} \right) \right] \\
- 30 \left[ \cos \left( \frac{\pi}{2} \right) - j \sin \left( \frac{\pi}{2} \right) \right] - 60 \left[ \cos \left( \frac{\pi}{4} \right) - j \sin \left( \frac{\pi}{4} \right) \right] \\
= [35.36 + j35.36] + [j30] + [-42.43 + j43.43] \\
= -7.07 + j107.78 = 108.013 e^{j1.6363}
\]

• The results agree