Frequency Response Example

An LTI system has impulse response
\[ h[n] = \delta[n] + 3\delta[n-1] + \delta[n-2]. \]

- Find the frequency response \( H(e^{j\hat{\omega}}) \)
- In particular find \( H(e^{j\hat{\omega}}) \) when \( \hat{\omega} = \pi/4 \)
- Is \( H(e^{j\hat{\omega}}) = 0 \) for any \( \hat{\omega} \in [0, \pi] \)?

- Given the impulse response, we know that the frequency response is just
  \[ H(e^{j\hat{\omega}}) = \sum_{n=0}^{M} h[n]e^{-j\hat{\omega}n} \]

- Here we have then
  \[ H(e^{j\hat{\omega}}) = 1 + 3e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} = e^{-j\hat{\omega}}[e^{j\hat{\omega}} + 3 + e^{-j\hat{\omega}}] = e^{-j\hat{\omega}}[2\cos(\hat{\omega}) + 3] \]

- At \( \hat{\omega} = \pi/4 \) we have
  \[ H(e^{j\pi/4}) = 1 + 3e^{-j\pi/4} + e^{-j\pi/2} = \left[ 1 + 3\cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{2}\right) \right] - j\left[ 3\sin\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{2}\right) \right] = \left[ 1 + \frac{3}{\sqrt{2}} \right] - j\left[ 1 + \frac{3}{\sqrt{2}} \right] = 4.414e^{-j0.7854} \]

- To consider if \( H(e^{j\hat{\omega}}) \) is zero anywhere on \([0, \pi]\) we can just look at the final form
  \[ H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}[2\cos(\hat{\omega}) + 3] \]

- We see that the term \( 2\cos(\hat{\omega}) + 3 \) can never be zero, so there are no \( \hat{\omega} \) values in this case where the filter gain is zero.