A rocket nozzle has to operate at an altitude of 25 km at a thrust of 5000 N. The chamber pressure and temperature can not exceed 2.068 MPa and 2800 K respectively. Assume the product species to have $\gamma = 1.3$ and $R = 355.4 \text{ J/kg K}$. Determine the exit velocity, exit Mach number and specific impulse of the rocket assuming $A_t = 0.00139 \text{ m}^2$.

**Given:**
- $P_0 = 2.068 \text{ Pa}$
- $T_0 = 2800 \text{ K}$
- $F_t = 5000 \text{ N}$
- $P_a = 0.00255 \text{ MPa (h = 25 km)}$
- $\gamma = 1.3$
- $R = 355.4 \text{ J/kg K}$
- $A_t = 0.00139 \text{ m}^2$

**Find:** $V_e$, $M_e$, Isp

Since the rocket is being designed to operate at only one altitude, the nozzle can be designed for ideal expansion.

$$P_e = P_a$$

$$\frac{P_0}{P_e} = \left(1 + \frac{\gamma - 1}{2} M_e^2 \right)^{\frac{\gamma}{\gamma - 1}}$$
\[
M_e = \left[ \frac{2}{\gamma-1} \left( \left( \frac{P_o}{P_e} \right)^{\gamma-1} - 1 \right) \right]^{1/2}
\]

\[
= \left[ \frac{2}{1.3-1} \left( \frac{2.068 \text{ MPa}}{0.00255 \text{ MPa}} \right)^{1.3-1/1.3} - 1 \right]^{1/2}
\]

\[\boxed{M_e = 4.96}\]

\[V_e = M_e a_e = M_e \sqrt{\gamma R T_e}\]

\[\frac{T_o}{T_e} = 1 + \frac{\gamma-1}{2} M_e^2\]

\[T_e = \frac{T_o}{1 + \frac{\gamma-1}{2} M_e^2}\]

\[= \frac{2800 \text{ K}}{1 + \frac{1.3-1}{2} (4.96)^2} = 598.9 \text{ K}\]

\[V_e = 4.96 \sqrt{1.3 (355.4 \text{ J/kg K}) (598.9 \text{ K})}\]

\[\boxed{V_e = 2,609.1 \text{ m/sec}}\]

\[I_{sp} = \frac{V_e}{g_0} \quad \text{(ideal only)}\]

\[\boxed{I_{sp} = 265.9 \text{ sec}}\]